Lab #8 - Electric Potentials and Fields - 850

Introduction: The objective of this experiment is to study the potentials, equipotential curves and electric fields produced by various electrostatic charge distributions. The potentials are measured in a two-dimensional tray of water which restricts the type of charge distributions to those that produce electric fields that are parallel to a single plane. Consequently, the potentials of a point charge cannot be measured accurately. However, the potentials of parallel plates, cylindrical capacitors and parallel line charges can be measured. The conditions in this experiment are not truly electrostatic since electric currents exist in the water. However, the resulting potentials are the same as for the exact electrostatic case.

Equipment: water tray, beaker, transparent grid, two mounted probes, various copper plates, DVM with included probes, two wires with banana plugs at one end and spade plugs at the other end (1 red, 1 black) and 1 black wire with banana plugs on each end.

Theory: Point charges produce potentials, V, and electric fields E that are easily calculated. However, the potentials for charge distributions and charges on conductors are more difficult to calculate since the charge distributions may be unknown. A curve along which the potential remains constant is referred to as an equipotential curve and is convenient for visualizing the potentials produced by charges or charge distributions. Since no work is done by the electric field when a charge moves along an equipotential curve the component of the electric field that is parallel to the path must be zero. Therefore, at all points on the equipotential curve the electric field is perpendicular to the equipotential curve as shown in Fig. 1a for a positive point charge. The components of E (in two dimensions) are related to V by:

\[ E_x = \frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x} \]
\[ E_y = \frac{dV}{dy} \approx -\frac{\Delta V}{\Delta y} \]  

where \( \Delta V \) is the small change in V associated with a small displacement \( \Delta x \) or \( \Delta y \). The standard units for E are V/m. Convert all of your measurements of V/cm in this lab to V/m to report your data.

Experimental: The equipment used to measure the potentials is shown in Fig. 1b. The water tray should have enough water to cover the copper plates placed in the tray. A plastic sheet with a rectangular coordinate system is located at the bottom of the tray as shown in the diagram. A sine wave potential is applied to the probes “A” and “B” which are used to construct different charge configurations. This type of potential minimizes electrolysis of the water which would be a problem with a DC supply.

- Connect Probe A to the Pasco Interface with a black spade to banana plug wire “c” going to the black output terminal of the Pasco Interface. Use a black banana to banana plug wire “b” to connect the ground terminal on the Pasco Interface to the ground terminal on the voltmeter. Use the red voltmeter wire “d” (inside the voltmeter) to connect to the positive voltage input terminal of the voltmeter. Finally connect the spade to banana plug wire “a” from the red output terminal on the Pasco Interface to probe “B” (Note: Fig. 1b does not necessarily show your particular voltmeter). With these connections a potential difference is applied between probes “A” and “B” to simulate a particular charge distribution while the voltage probe measures the potential difference between the “voltage” probe and probe “A”. Notice that probe “A” is at system ground (0V) potential.
- Set up the AC voltmeter by rotating the large knob to the V~ position, indicating AC voltage.
- The peak-to-peak amplitude of the sine wave potential difference generated by the Pasco Interface is \( V_0 \). However, the voltage that is measured by the AC voltmeter is a RMS (root mean square voltage) \( \frac{V_0}{\sqrt{2}} \).

(See appendix C for the explanation). That means that a 5V (peak-to-peak) sine wave will read as 3.536V on the AC voltmeter. To make the readings easier, we will set the
The potential at any point “x” between the plates is:

\[ V(x) = \int_{0}^{x} \vec{E} \cdot d\vec{s} = \int_{0}^{x} \frac{\sigma}{\varepsilon_0} \cdot d\vec{i} = 30x \]  

The electric field \( E_x \) is the negative derivative of the potential. For example, an approximate value for the electric field at the location \( x = 1.25 \) (cm) is obtained from Eq.1:

\[ E_x \approx -\frac{\Delta V}{\Delta x} = -\frac{V(1.5) - V(1.0)}{0.5} \left( \frac{V}{cm} \right) \]  

The goals of this part of the experiment are to: (i) measure the components of the electric field between the capacitor plates and determine the direction of the field, (ii) map one of the equipotential curves and (iii) measure the potentials and fields along the X axis shown in Fig. 2.

(i) Components of the Field:

- Align the copper strip in the tray with the inside edge of one strip along the Y axis (at \( x = 0 \)) and the inside of the other strip at \( x = 10.0 \) (cm). Place probe “A” and Probe “B” in contact with the left and right strips respectively.
- Add 500 ml of water (about 1 cup) into the tray.
- Check to see that the equipment has been set up correctly by placing the voltmeter probe in contact with the left strip. The potential should be zero volts. With the voltmeter probe in contact with the right strip, the potential should be very close to 3.00 volts.
- Also check that the water is covering the plates. Using the voltmeter probe measure and record the potentials \( V_a, V_b, V_c \) and \( V_d \) at the locations: \( a \) (6,0), \( b \) (4,0), \( c \) (5,1) and \( d \) (5,-1) cm. The electric field components at \( P \) (5,0,0) (cm) are obtained from Eq. (1):

\[ E_x = -\frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x} = -\frac{V_a - V_b}{2} \left( \frac{V}{cm} \right) \]

\[ E_y = -\frac{dV}{dy} \approx -\frac{\Delta V}{\Delta y} = -\frac{V_c - V_d}{2} \left( \frac{V}{cm} \right) \]

- Compute the field components \( E_x \) and \( E_y \) at the location “P” and determine the angle “\( \theta \)”. Is the direction of the measured electric field consistent with the expected direction for an ideal, very large capacitor?

We obtained the field components by replacing the differential quantities “dV”, “dx” and “dy” by relatively large values of “\( \Delta V \)”, “\( \Delta x \)” and “\( \Delta y \)” which works well for large parallel plate capacitors where the field is actually uniform. In other situations, it would be necessary to use much smaller values for “\( \Delta V \)”, “\( \Delta x \)” and “\( \Delta y \)”.

(ii) Equipotential Curve:

The equipotential surfaces for an ideal, very large capacitor are planes parallel to the conducting plates. The intersection of these planes with the water layer produces equipotential lines in the “XY” plane. Since our capacitor is not infinitely large we might expect that the equipotential lines will not be exactly the same as the lines for an ideal, large capacitor. Any differences would be most noticeable near the ends of the plates and are referred to as “end effects”. We will measure one equipotential curve to

In this window select the following:
- Sine waveform,
- Frequency 100Hz.
- Sweep Type off,
- Amplitude 4.243,
- Voltage limit 4.243 V,
- Current limit 0.55 A.
- ON button selected

The ON button must be selected so that the generator will run continuously for the entire experiment. Do not record data with the Capstone program. Only enter readings into Excel. Check that the AC voltmeter is reading 3.00V when the voltmeter probe is touching Probe B.

A) Parallel Plate Capacitor

Theory: For an ideal parallel plate capacitor having very large plates the electric field is perpendicular to the plates and is uniform. Since the fields are perpendicular to the equipotential surfaces, the equipotential surfaces should be planes located parallel to the plates. The electric field between the plates of a very large parallel plate capacitor can be computed using Gauss’ Law. If the surface charge density on the plates has a magnitude of “\( \sigma \)” and the plate at “\( x = 0 \)” has the negative charge, then the electric field is:

\[ \vec{E} = \left[ \frac{\sigma}{\varepsilon_0} \right] \hat{i} \]

We choose the potential to be zero at “\( x = 0 \)” and let \( \Delta V_0 \) (3.0 V) be the potential difference between the plates separated by 0.1m.:

\[ \Delta V = \int_{0}^{0.1} \frac{\sigma}{\varepsilon_0} \cdot dx = \frac{\sigma}{\varepsilon_0} \cdot x = \frac{\sigma}{\varepsilon_0} \cdot 0.1 = 30(\text{V/cm}) \]

\[ \vec{E} = -\frac{\sigma}{\varepsilon_0} \hat{i} = -30\hat{i}(\text{V/cm}) \]  

(1)

The potential at any point “\( x \)” between the plates is:

\[ V(x) = \int_{0}^{x} \vec{E} \cdot d\vec{s} = \int_{0}^{x} \frac{\sigma}{\varepsilon_0} \cdot d\vec{i} = 30x \]  

(2a)

The electric field \( E_x \) is the negative derivative of the potential. For example, an approximate value for the electric field at the location \( x = 1.25 \) (cm) is obtained from Eq.1:

\[ E_x \approx -\frac{\Delta V}{\Delta x} = -\frac{V(1.5) - V(1.0)}{0.5} \left( \frac{V}{cm} \right) \]  

(2b)
determine if the ideal capacitor model works well and possibly observe “edge effects”.  

- Place the voltmeter probe at (3.0, cm) and note the potential. Move the voltmeter probe up to the line “y = 1 cm” and by moving the probe slowly along this line, parallel to the “X” axis, find the “x” coordinate of the location that has the same potential that you just observed. Record the coordinates of this point which will be on the same equipotential that passed through (3.0, cm). Repeat this step, to find more points on this curve, by moving the probe to the following lines (y = 2, 3, 4, 5, 6, 7, 8 and 9 cm). For each of these “y” coordinates find the corresponding “x” coordinate of the location that has the same potential you originally observed. Record these coordinates in an Excel worksheet.  

- Plot this equipotential curve. In Excel, change the scale limits for both the X and Y axes to be 0 and 10 cm. How does this curve compare with the expected result for an ideal, large capacitor? Is there any evidence of “edge effects”? Explain.  

(iii) Measurement of the Potentials and Fields along the X Axis:  

- Measure and record the potential with the voltmeter probe as it is moved from (0.5,0) to (9.5,0) cm in 0.5 cm increments. The potential should increase as you move away from probe A towards probe B. If the opposite occurs, your wiring is incorrect.  

- Enter your values of “X” and the potential V(x) into two columns of an Excel worksheet. Construct a third column for X’ which is the value midway between the locations where V was measured (X’ = X + 0.25) (cm). Find the value for the electric field at X’ in a fourth column using the method illustrated by Eq. (2b). Assume, for example, that X is in column A, potential is in column B, X’ in column C and the computed field is in D. The first electric field at X’ = 0.25 (cm) would be obtained by using the following formula in cell D2: “= (B3-B2)/.5”. It has been assumed that column titles are in the first row and the actual data starts in the second row.  

Using this procedure obtain the fields and then find the mean value of the field using the AVERAGE function. You may need to exclude the first and last values from the average. Express the answer in Volts/Meter.  

- Plot E, versus X’. Set the upper limit for the X axis to be zero for this plot. Is your plot consistent with Eq. (1)?  

- Plot V versus X. Is this graph consistent with Eq. (2a)? From a fit of this graph determine the electric field. Compare this value with the mean of the computed values of E.  

- Find the percent error between your measured values of the electric field and the theoretical value of 30.0 (V/m) obtained from Eq. (1) for a large parallel plate capacitor.  

- Measure the potential at a number of points on the conducting surfaces. What do you conclude? What are the X and Y components of the electric field on the surface of the conductor?  

B) Cylindrical Capacitor:  

Theory: Figure 3 shows a cross section of a cylindrical capacitor. The radii of the inner and outer cylindrical conductors are R1 and R2 respectively. If the simulated charge per unit length on the inner conductor of radius R1 is -λ, then by Gauss Law:  

$$\bar{E} = -\frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(3)}$$  

where the unit vector is in the radial direction.  

The difference in potential between the inner conductor at zero potential and a point at the radial distance “r” is:  

$$\Delta V = V(r) - V(R_1) = V(r) - 0 = -\int_{R_1}^{r} \bar{E} \cdot d\bar{r}$$  

We integrate this equation to find the potential V(r) at the distance “r”:  

$$V(r) = -\frac{\lambda}{2\pi\varepsilon_0} \ln R_1 - \frac{\lambda}{2\pi\varepsilon_0} \ln r \quad \text{(4)}$$  

In the above equation we have used the fact that the potential at R1 is zero.  

Since the potential depends directly on the radial distance “r”, the potential will be the same at all locations on a cylindrical surface having this radius. Therefore, the equipotential curves in the “XY” plane are circles concentric with Probe A.  

The goals of this part of the experiment are to: (i) map one of the equipotential surfaces, (ii) measure the potential between the cylindrical surfaces of the capacitor and compare with the theoretical model and (iii) obtain the electric field and compare with the theoretical model.  

(j) Equipotential Curve:  

- Construct a cross-section of a cylindrical capacitor by placing the copper ring with its center exactly at (0,0). Place probe A (ground) at (0,0) and probe B on top of the copper ring.  

- Measure, in the first quadrant only, the equipotential curve that passes through the location (0, 3 cm). Begin by noting the potential at that point. Move the voltmeter probe over to the line “x = .5 cm” and by moving the probe slowly vertically from this point, find the “y” coordinate of the location that has the same potential that you just observed. Record the coordinates of this point which will be on the same equipotential that passes through (0, 3 cm). Repeat this step by moving the probe to the following lines (x = 1.0, 1.5, 2.0, 2.5 and 3.0 cm). For each of these “x” coordinates find the corresponding “y” coordinate of the location that has the same potential you originally observed. Record these coordinates in an Excel worksheet.  

- Plot this equipotential curve. Change the scale limits for both the X and Y axes to be 0 and 4.0 cm. How does this equipotential curve compare with the expected result for cylindrical capacitor?
On the same graph plot the theoretical equipotential curve that passes through (0, 3.0) cm which is, of course, a quarter circle with center at the origin and having a radius of 3.0 (cm). To plot this curve, create a column for “x” in the range between 0 and 3 (cm) with an increment of 0.1 (cm). Use the equation of the circle to find the corresponding values of “y” on the circle. Is your measured curve consistent with the curve predicted by our model?

(ii) Potentials:
- Measure and record in an Excel worksheet, the potential at radial increments of 0.5 cm from (0,0) to the surface of the ring.
- The distance r’ is the distance which is obtained by adding 0.25 (cm) to the values of r. (see fig 8.)

![Fig 8.](image)

- Plot V(r) versus the natural logarithm of r (not r’), and do a linear fit. (The natural logarithm function in Excel is ln().) Is your plot consistent with Eq. (4)?

(ii) Electric Fields:
- Using the method outlined in Part A(iii), find experimental values for the electric field at r’ by computing the correct potential differences.
- Plot a graph of the absolute value of the field versus the distance r’ and do a power fit. Compare your fitted power with the value of minus one predicted by Gauss’s Law.

C) Long, Parallel, Oppositely Charged Wires:
The goals of this part of the experiment are to: (i) measure the potential along the line connecting two long parallel wires and (ii) to map two equipotential curves and compare with the predictions of a model. In parts A and B of the experiment the field lines were nearly confined to the region between the conducting surfaces and the finite size of the tray would have an insignificant effect on the fields and potentials. However, for the long parallel line charges the fields extend to infinity and the limited size of the tray may be significant particularly for fields near the tray’s edge.

(i) Potential between the Wires:
The wires having a radius “R” (9.9x10⁻² cm) are perpendicular to the XY plane and are separated by the distance “d” (10 cm). The wire passing through the origin has a negative linear charge density -λ, while the other wire has the positive charge density +λ. The potential difference between the origin and a point P at (x,0) is:

\[ \Delta V(x) = \frac{\Delta V_0}{2} \left( 1 + \frac{x}{d-x} \ln \frac{d-R}{R} \right) \]  

In this equation \( \Delta V_0 \) is the difference in potential between the two wires and \( \Delta V(x) \) is the potential difference recorded by the voltmeter when the probe is at the location “x”. The details of this calculation are given in Appendix A.

- Place the Probe A at (0,0) and Probe B at (10,0) cm. The origin will be at the location of the left wire. This wire is at zero potential simulating the negatively charged wire. Measure the potential in increments of 0.5 (cm) from Probe A to Probe B, starting at 0.5 (cm). Since the voltmeter is connected between the left probe and the volt meter probe, the measured potential difference will be \( \Delta V(x) \).

- In an Excel worksheet enter the nominal value of 9.9x10⁻² (cm) or (9.9E-4 m) in cell B1, the value of 10 (cm) for “d” in cell B2 and finally 3.00 (V) for \( \Delta V_0 \) in cell B3. In cell B4 we calculate the constant logarithmic term in the denominator of eq. (5) by entering the formula: 
  \[ =LN((B$2-B$1)/B$1) \]

- Enter your measured values of “x” (cm) and \( \Delta V(x) \) in columns “A” and “B” respectively starting with the first data in row 10. In column “C” calculate the logarithmic nominator term in Eq. (5) by entering, in cell C10, the formula:
  \[ =LN((A10/(B$2-A10))) \]

Finally, in cell “D10”, calculate the theoretical value of \( \Delta V(x) \) from Eq. (5) using the formula:

\[ =0.5*B$3*(1+C10/B$4) \]

- In cell “E10”, compute the square of the difference between the theoretical and measured potentials. We will use Solver to minimize the sum of the squares of these differences. Therefore, create a cell for this sum.
- Plot a graph of the measured potential difference \( \Delta V(x) \) versus “x” in meters. Plot the data points but do not connect with lines. Plot the theoretical potentials on the same graph.
- Since the values of R, d and \( \Delta V_0 \) have some uncertainty, we may be able to improve our model by having Solver find better values that are chosen to minimize the sum of the differences squared. Use Solver to minimize the sum of the differences squared by letting it change R, d and \( \Delta V_0 \). You may find that it is necessary to include a constraint to limit the value of “R” to positive values as shown in the figure. Does the Solver solution improve the agreement between the measured and model values? Is a...
model of two long, parallel conducting wires reasonably consistent with your data?

(ii) Equipotential Curve:
The calculation of the equipotential curves of the parallel line charges is simplified if we place the line charges passing through the “XY” plane at (-d,0) and (+d,0) with the origin mid-way between as shown in Fig. 5. The line charges have opposite sign, are very thin and separated by a distance “2d”. Note that “d” is not the same as in the previous part. As shown in Appendix B, the equipotential curve that passes through the point P’ at (x’,0) on the X axis (with x’ > d) is a circle of radius “R” with its center at (X_c,0). The equation of this circle is:

\[(x - X_c)^2 + y^2 = R^2 \quad \ldots \ (6)\]

The values for the X coordinate of the center and the radius are:

\[X_c = \frac{d^2 + x'^2}{2x'} \quad ; \quad R = \frac{x'^2 - d^2}{2x'} \quad \ldots \ (7)\]

For the case of the equipotential curve that passes through a point P” (x”,0) which is to the left of the line charge (x” < d), the X coordinates of the center and the radius are:

\[X_c = \frac{d^2 + x'^2}{2x''} \quad ; \quad R = \frac{d^2 - x'^2}{2x''} \quad \ldots \ (7a)\]

A few of the theoretical equipotential curves obtained from Eq. (6) are shown in Fig. 7 of the Appendix. Eq. (7a) predicts that the equipotential curve passing through the origin will have an infinite radius and therefore be a straight line along the “Y” axis.

Equipotential Curve Passing Through the Origin:
- Use the same experimental arrangement as in part C(i) but place the left probe at (-5,0) and the right probe at (+5,0) cm (so that “d” in Fig. 5 is 5 cm). In this part of the experiment, any time variation in the potential caused by factors such as fluctuations in the applied potential or conductivity of the water can make it difficult to obtain good equipotential curves. For this reason, you need to work carefully but rapidly to achieve the best possible results. Place the voltmeter probe at the origin and note the potential. Quickly move the voltmeter probe up to the line “y = 1.0 cm” and by sliding the probe along this line find the “x” coordinate of the location that has the same potential that you just observed. Record the coordinates of this point which will be on the same equipotential that passed through the origin. Repeat this step by moving the probe to each of the following lines (y = 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, and 8.0 cm). However, between each measurement, measure again the value of the potential at the origin and use this new value in the next step. For each of these “y” coordinates, quickly find the corresponding “x” coordinate of the location that has the same potential you just observed. Record these coordinates in an Excel worksheet.

- Plot this equipotential curve. In Excel, change the scale limits for both the X and Y axes to be 0 and 10 (cm). How does this equipotential compare with the predictions of our model?

Equipotential Curve Passing Through (8.0, 0.0 cm):
- Place the voltmeter probe at (8.0,0 cm) and note the potential. The value of ‘x’ in Fig. 5 is 8.0 (cm).
- Move the voltmeter probe over to the line “x = 7.5 cm” and by sliding the probe along this line find the “y” coordinate of the location that has the same potential that you just observed. Record the coordinates of this point which will be on the same equipotential that passed through the origin. Repeat this step by moving the probe to each of the following lines (x = 7.0, 6.5, 6.0, 5.5, 5.0, 4.5, 4.0 and 3.5 cm). However, between each measurement, measure again the value of the potential at (8.0, 0.0) and use this new value in the next step. For each of the “x” coordinates find the corresponding “y” coordinate of the location that has the same potential you just observed. Record these coordinates in an Excel worksheet.

- Plot this equipotential curve. In Excel, change the scale limits for both the X and Y axes to be 0 and 10 (cm).
- Using Eq. (7) with the values of 5.0 and 8.0 (cm) for d and x respectively we find the radius “R” of the equipotential circle is 2.44 (cm) and the “x” coordinate “X_c” of the center of the circle is 5.56 (cm).
- In another area of your worksheet construct a column of values for “x” starting at 8.0 (cm) and ending at 3.5 (cm) with an increment of -0.1 (cm). For each value of “x” use Eq. (6) to calculate the corresponding values of “y”. Use the positive root. Plot this theoretical model curve in the first quadrant only.
- Compare the measured equipotential curve with the theoretical model prediction.

Appendix A: The Potential between Two Long Charged Cylinders
The first objective is to find the total field E produced by both line charges shown in Fig. 6 at a point P(x,0).

The charge distribution on each wire is altered by the presence of the other wire. However, since “R” is much less than “d” the charge distribution remains reasonably uniform and we can use Gauss’s Law to find the electric field a distance “r” from each of the long straight wires:
\[ \vec{E} = \frac{\lambda \hat{r}}{2\pi \epsilon_0 r} \]  
\hspace{1cm} (8)

Considering the sign of the charges the total field at P is:
\[ \vec{E} = -j \frac{\lambda}{2\pi \epsilon_0} \left( \frac{1}{x} + \frac{1}{d - x} \right) \]  
\hspace{1cm} (9)

The potential difference \( \Delta V(x) \) between point P and the surface of the wire at the origin is:
\[ \Delta V(x) = -\int_{\infty}^{x} \vec{E} \cdot ds = \frac{\lambda}{2\pi \epsilon_0} \int \left[ \frac{1}{x} + \frac{1}{d - x} \right] dx \]
\hspace{1cm} (10)

If the potential difference between the two wires is \( \Delta V_0 \) then we can use the known difference in potential \( \Delta V_0 \) between the wires to eliminate the unknown charge density \( \lambda \). Using \( x = d - R \) in Eq. (11) gives:
\[ \Delta V_0 = \frac{\lambda}{2\pi \epsilon_0} \left( \ln \frac{d - R}{R} + \ln \frac{d - R}{R} \right) \]
\[ \frac{\lambda}{2\pi \epsilon_0} = \frac{\Delta V_0}{2 \ln \frac{d - R}{R}} \]  
\hspace{1cm} (11)

Combining equations (10) and (11):
\[ \Delta V(x) = \frac{\Delta V_0}{2} \left\{ 1 + \frac{\ln \frac{x}{d - x}}{\ln \frac{d - R}{R}} \right\} \]  
\hspace{1cm} (12)

**Appendix B: Equipotentials of Parallel Line Charges:**

From Gauss’ Law the field of a long line charge is:
\[ E = \frac{\lambda}{2\pi \epsilon_0 r} \]

The potential at a distance “\( r_1 \)” from the positive line charge is found by integrating the electric field from a reference point (where the potential is zero) to a distance “\( r_1 \)” from the charge:
\[ V_1 = +\frac{\lambda}{2\pi \epsilon_0} \ln r_1 \]

The potential of the negative line charge at a distance of “\( r_2 \)” from it is:
\[ V_2 = -\frac{\lambda}{2\pi \epsilon_0} \ln r_2 \]

These potentials could also include a constant term that depends on the reference position. However, we choose the reference position where the potential is zero, to be along the Y axis where \( r_1 = r_2 = s \). The total potential at any point is the sum of these two potentials. The total potential on the Y axis is zero:
\[ V(0, y) = V_1 + V_2 = +\frac{\lambda}{2\pi \epsilon_0} \ln s - \frac{\lambda}{2\pi \epsilon_0} \ln s = 0 \]

The total potential at any point “P” is:
\[ V = V_1 + V_2 = +\frac{\lambda}{2\pi \epsilon_0} \ln r_1 - \frac{\lambda}{2\pi \epsilon_0} \ln r_2 = -\frac{\lambda}{2\pi \epsilon_0} \ln \left( \frac{r_1}{r_2} \right) \]

**Fig. 6**

At all points on the equipotential curve that passes through “P” the natural log of the ratio of the radii must be the same. Therefore, on an equipotential curve we have:
\[ \frac{r_1}{r_2} = c \]

where “c” is a constant. We can find the equation of the equipotential curve passing through “P”.

From the right triangles of Fig. 6 we have:
\[ r_1^2 = (x - d)^2 + y^2 \]
\[ r_2^2 = (x + d)^2 + y^2 \]
\[ \frac{(x - d)^2}{c^2} + \frac{y^2}{c^2} = x^2 + 2dx + d^2 + y^2 \]
\[ \frac{(x + d)^2}{c^2} = x^2 + 2dx + d^2 + y^2 \]
\[ x^2 - 2d\left(\frac{c^2 + 1}{1 - c^2}\right)x + y^2 = -d^2 \]

By completing the square term in “x” we see that the above equation is a circle:
\[ x^2 = \frac{2d(c^2 + 1)}{1 - c^2} x + d^2\left(\frac{c^2 + 1}{(1 - c^2)^2}\right)y^2 = -d^2 + \frac{d^2(c^2 + 1)^2}{(1 - c^2)^2} \]
\[ \left( x - \frac{d(c^2 + 1)}{1 - c^2} \right)^2 + y^2 = \left( \frac{2dc^2}{1 - c^2} \right)^2 \]  
\hspace{1cm} (13)

This is the equation of a circle with its center at (X,0) and a radius “R” which are:
\[ X = \frac{d(1 + c^2)}{1 - c^2} \]  
\[ R = \frac{2cd}{1 - c^2} \]
Fig. 5 shows an equipotential curve that passes through the point P' which is on the X axis at (x',0). The coordinate x' is:

\[ x' = X_c - R = \frac{d(1-c)}{1+c} \]  

(14)

The equipotential that passes through P' has a unique value of “c” that can be obtained from Eq. (14):

\[ x' = \frac{d(1-c)}{1+c} \Rightarrow c = \frac{d - x'}{d + x'} \]  

(15)

With this value of “c” the radius and Xc are:

\[ X_c = \frac{d^2 + x'^2}{2x'} \quad ; \quad R = \frac{x'^2 - d^2}{2x'} \]  

(16)

If we wanted the equipotential curve that passes through a point P"(x",0) which is to the left of the line charge (x" < d), the X coordinate of the center and the radius are:

\[ X_c = \frac{d^2 + x'^2}{2x'} \quad ; \quad R = \frac{x'^2 - d^2}{2x'} \]  

The equipotential curve through P’ is found using Eq.(13) and the values for “R” and “Xc” from Eq. (16). A plot of several of the equipotentials obtained from this model is shown in Fig. 7.

**Appendix C: The Root Mean Square Potential.**

The sine potential has an amplitude “V_0” (zero to peak value), an angular frequency “ω” and a period “T” which is \( \frac{2\pi}{\omega} \). The potential at a time “t” is:

\[ V(t) = V_0 \sin(\omega t) \]

We calculate the average value of the square of the potential over a time interval equal to the period. This average is:

\[ (V^2)_{ave} = \frac{\int_0^T V_0^2 \sin^2(\omega t) dt}{T} \]

\[ (V^2)_{ave} = \frac{V_0^2}{2\omega} \int_0^\pi \sin^2(\omega t) dt \]

\[ (V^2)_{ave} = \frac{V_0^2}{2\omega} \left[ 0 - \frac{\sin(2\omega) - 4}{4} \right] \]

\[ (V^2)_{ave} = \frac{V_0^2}{2\omega} \left[ \frac{\omega}{2} - \frac{\sin(2\omega)}{4} \right] \]

The root mean square value is:

\[ V_{rms} = \sqrt{\frac{V_0^2}{2}} = \frac{V_0}{\sqrt{2}} \]

If we want \( V_{rms} = 3.000 \), then \( V_0 = 4.243 \) volts.