Lab #12 - Electromagnetic Induction II - 850

Equipment: Pan balance, Copper pipe, cylindrical magnet assembly, mass set, vertical support bar with table clamp, three-finger clamp, Pasco RCL experiment board with iron cylinder a short aluminum cylinder, two medium length wires with banana plugs at both ends, a multi-meter, and motion sensor.

Introduction: Faraday induction, eddy currents and magnetic braking are studied in Part A. In Part B of this experiment Faraday's Law is investigated using a square wave potential applied to a coil in series with a resistor.

Part A: Electromagnetic Induction and Eddy Currents: Professor McGinn originated this part of the experiment and contributed to its design and development. The motion of a magnet that is allowed to fall through a non-magnetic, conducting, cylindrical copper tube is studied. The magnet experiences a resistive magnetic drag force caused by electric currents induced in the tube by the magnet. This drag force depends on the first power of the magnet's velocity. When the magnet reaches a speed where the drag force becomes as large as the gravitational force the magnet stops accelerating and it moves with a constant terminal speed. A magnet moved through a single wire loop creates a current in that loop by Faraday induction as found in the previous experiment. When a magnet is passed through a conducting cylindrical tube currents are also induced, however they are more difficult to visualize. These currents are called “eddy currents” and are very important in a number of technological applications such as stovetops with Faraday induction heating by eddy currents in the frying pan, magnetic brakes and transformers.

Theory: A brief description of the induced resistive force is given now. However, more details can be found in the Appendix of this experiment. In Fig. 1 the magnet is falling down through the tube and the assumed direction of the field is shown. The field of the magnet decreases as we move away from the magnet. A particular cross section of the tube is shown in the figure as the red circle which is also labeled “eddy current”. The magnetic flux through this circle is increasing since the magnet is approaching. The rate of change of this flux is directly proportional to the speed of the falling magnet. According to Lenz’s Law an induced eddy current will flow in the direction shown by an arrow in this circle. Eddy currents will also flow in all other cross sections and will be larger as we get closer to the magnet. The direction of the induced magnetic field will be in the direction opposite to the field of the magnet. As shown with more detail in the Appendix, this field exerts a net upwards, or resistive, force on the falling magnet which is proportional to the speed of the magnet. Above the magnet, the field is in the same direction but produces a decreasing flux since the magnet is moving away from a particular eddy loop. The induced current above the magnet circulates in the opposite direction. However, this current also exerts an upwards force on the falling magnet. The total resistive force on the moving magnet is:

\[ F_{res} = cv \]  \hspace{1cm} (1)  

The units of ‘c’ are (kg/sec)

where “v” is the speed and “c” is a constant that would be very difficult to calculate as explained in the Appendix. We can apply Newton’s II Law to the motion of the magnet and connected “hanger” which have a total mass “m”:

\[ mg - cv = ma \rightarrow mg - cv = m \frac{dv}{dt} \]  \hspace{1cm} (2)

If the magnet falls from rest the induced resistive force is initially zero but rapidly increases until it is as large as the gravitational force and we reach the terminal speed “\( v_t \)” where the acceleration is zero:

\[ mg - cv = 0 \rightarrow v_t = \frac{mg}{c} \]  \hspace{1cm} (3)

If at some time “\( t_0 \)” the speed is “\( v_0 \)” then the solution of Eq. 2 gives the velocity of the magnet at a later time “\( t \)”:

\[ v(t) = \frac{mg}{c} - \left( \frac{mg}{c} - v_0 \right) e^{-\frac{c}{m}(t-t_0)} \]  \hspace{1cm} (4)

The maximum value of the speed is found by letting the time become sufficiently large. The limiting value of the speed is the terminal speed, “\( v_t \)” which is “\( mg/c \)”.

Experiment:

• Plug in and select the motion II sensor with the hardware setup icon. Set the motion sensor switch to the wide-angle position and select a data collection speed of 40 measurements per second (you may have to reduce the data collection rate to
rubber band around one of the clamp fingers and pass it through the string loop that is connected to the magnet. Loop the other end of the rubber band around one of the opposite fingers. The rubber band should cross over the top of the tube and through the string loop so that the falling loop will be stopped by the rubber band when the magnet is near the bottom of the tube. Before you collect your first data set, be sure the rubber band passes through the upper loop of string so that the string will be caught by the rubber band and prevent the hanger from hitting the motion sensor.

(i) Dependence of Terminal Velocity on the Total Mass:
- Our model (Eq. 3) predicts that the terminal velocity should increase linearly with the total mass of the magnet and the hanging mass “m”:

\[ v_t = \left(\frac{g}{c}\right) m \]

Begin with no added mass on the hanger. Pull the top string up until the magnet is about five cm below the top of the tube. If the hanger is swinging from side to side bring it to rest. Start collecting data and then release the magnet after you hear “clicks” from the motion sensor. If everything is set up correctly data collection should stop automatically after the magnet comes to rest. The plot of speed versus time should be reasonably smooth and show that the speed approaches a terminal speed as found in the falling balloon experiment. If your data is not relatively smooth, try realigning the motion sensor, changing the wide/narrow switch of the sensor and possibly reduce the data collection speed to get acceptable data. Highlight your data in the area of the terminal velocity and use the mean value function under the zoom icon to find the terminal speed. Record this value and the added mass.

- Do additional trials with added masses of 20, 40, 60, 80 and 100 grams. Only record your best estimate of the terminal velocity for each hanging mass. Using an Excel worksheet, plot the measured terminal velocity versus the total mass in kilograms. Is your data consistent with our model? Do a linear fit through the origin. From the slope \((g/c)\) of the fitted line determine an estimate for the parameter “c”.

(ii) Data and Model Comparison with Solver:
The objective of this part of the experiment is to compare the experimental speeds of the falling magnet with the model speeds calculated from Eq. 4. In order to make this comparison, we only need a single smooth data set with a mass of 100 grams added to the hanger. Do a few trials to obtain the smoothest data set possible. Begin with the motion sensor set for 40 Hz. If you can get smooth data at this speed, try a higher collection rate since more data will give a better fit provided that the data remains relatively smooth.
When you have obtained a good data set transfer the data to a Data Table. Copy all of the data to an Excel worksheet, reserving the first six rows for variables to be used by Solver. In cell $B$1 enter as a guess for "c" the value you found in part (i). In cell $B$2 enter the total mass of the magnet, hanger and 100 gram added mass in kilograms.

The small speeds that occur just after the magnet is released are not very accurately determined. Therefore examine your speeds and select the first speed that is larger than 0.15 (m/s). Let this speed be "v₀" and the corresponding time be "t₀". Enter your values of "t₀" and "v₀" in cells $B$3 and $B$4 respectively. Delete all the velocities and times before "t₀" from your worksheet. Also delete all velocities and times at the end of the table that occurred as the magnet was being stopped by the rubber band. Then cut and paste your data so that the time is in column “A” with the velocity in “B” and the first data pairs in row 10.

Use Eq. 4 to calculate (using the absolute addresses, $B$1, $B$2, $B$3 and $B$4) the model value for the speed in the column “C” after the measured speeds. The formula you should enter in cell C10 is given below. You can copy this formula (using the SELECT tool in Acrobat) and paste directly into the Excel cell.

\[ \text{Speed} = \frac{9.8 \times \text{Mass}}{\text{Time}} \]

In the next column find the square of the differences between the measured and model speeds. Place the value of the sum of the squares of these differences in a convenient cell.

- Use Solver to find the best values of the single parameter “c” by letting Solver change only the cell $B$1 that contains the parameter “c” (but not the values of “m”, “v₀” or “vₐ”) to minimize the sum of the squares of the differences.

Plot on the same graph the experimental and model values of the speed versus time. How well does our model represent the actual measured speeds of the falling magnet?

Using your Solver value of “c” and the total mass “m” calculate the terminal velocity and compare with your observed value.

How do the values of “c” determined in this and the previous method compare? Find their percent difference. Make sure all values are in SI (MKS) units or the value of c will be incorrect.

Part B: Faraday's and Lenz's Laws Applied to a Circuit with a Resistor and Coil in Series

Objective: The objective of this part of the experiment is to study electromagnetic induction by applying a square wave potential to a circuit having a resistor and coil in series. Without electromagnetic induction, Ohm's Law would predict that the current in the circuit would also be a square wave since for a resistor the applied potential and the current are directly proportional. However, with electromagnetic induction the sudden change in potential which occurs for a square wave causes a change in current and magnetic field inside the coil. This changing field causes a changing magnetic flux that, by Faraday’s Law will produce an induced \( \text{Emf} \). By Lenz's Law this \( \text{Emf} \) produces an induced current whose direction depends on whether the original current is decreasing or increasing.

Theory (i):

The Induced Current when the Applied Potential Suddenly Decreases to Zero:

Fig. 3 shows a circuit that includes a resistor, coil and a signal generator. Suppose the signal generator is applying a constant 5 volt potential difference across the coil and resistor. The current will be constant and the magnetic field produced in the coil will also be constant since it depends directly on the current in the coil wires. The current is assumed to be flowing in the direction of the arrow and since the flux through the coil is constant Faraday’s Law implies that there is no induced \( \text{Emf} \) in the coil wires. However, if the potential difference applied by the generator is a positive square wave as shown in Fig. 4, the applied potential will suddenly become zero at some instant. Without induction effects, the current would also become zero immediately as shown by the dotted current curve. However, in the coil the decreasing current produces a decreasing field and therefore a decreasing flux. According to Faraday’s Law there must be an induced \( \text{Emf} \) in the coil wires (while the flux is changing) and by Lenz's Law the induced current in the circuit flows in the same direction as the original current in order to oppose the decreasing flux. As a result of the induced current, we will find that the total current follows the solid curve shown in Fig. 4.

The magnetic field "B" in the coil is:

\[ B = \mu_0 n I \]

where "I" is the current and "n" is the number of turns per unit length. If the coil length is "d", the flux through all of the coils is:

\[ \Phi = ndAB = \mu_0 n^2 dAI = LI \quad (5) \]

where "A" is the cross sectional area of the loops. This result shows that the flux is proportional to the current. For convenience we have replaced \((\mu_0 n^2dA)\) by the constant "L" which is called the "self-inductance" of the coil and is measured in units called "henries". The flux is therefore LI. The induced \( \text{Emf} \) in the circuit is obtained from Faraday's Law:
\[ Enf = -\frac{d\Phi}{dt} = -L \left( \frac{dl}{dt} \right) \] ...(6)

Since the current is decreasing (dI/dt) is negative and -L(dI/dt) will be positive and represents the magnitude of the potential difference across the inductor. Since the inductor is acting like a battery to produce an induced current in the direction of the arrow, the high and low potential sides of the inductor are "b" and "a" respectively as shown in the figure. Recall that inside any source of \( Enf \) such as a battery the current flows from the low to high potential. The high and low potential sides of the resistor are "c" and "d" respectively. The current flowing in the circuit at a time "t" after the applied potential difference has become zero is "I(t)". Applying Kirchoff's Law we proceed counterclockwise around the circuit of Fig. 3 giving:

\[ \begin{align*}
0 + \left( -L \left( \frac{dI(t)}{dt} \right) \right) - RI(t) &= 0 \\
\text{where } V_{\text{sig gen}} &= 0. \text{ Separation of the variables in this equation produces:}
\end{align*} \]

\[ \frac{dI}{I} = -\frac{R}{L} \frac{dt}{dt} \]

If the left side of this equation is integrated from a zero current (at \( t = 0 \)) to \( I(t) \) (at any later time "t") and the right hand side from \( t = 0 \) to any later time "t" we obtain:

\[ I(t) = \frac{V_0}{R} e^{-\frac{t}{\tau}} \]

where \( \tau = L/R \) and \( V_0 \) is the potential difference across the resistor and coil at the instant \( t = 0 \), \( R \) is the total resistance in the circuit including the resistance of both the resistor and coil and "\( \tau \)" is "L/R", the time constant. This equation predicts that the current will not decrease to zero immediately after the applied potential difference is reduced to zero due to Faraday induction. The current will be reduced by the factor of \( (1/e) \) in a time interval "\( \tau \)". This decreasing current is also shown in Fig. 4 for the falling edge.

Theory (ii)
The Induced Current when the Applied Potential Suddenly Increases from Zero to \( V_0 \):
Since the current is increasing (dI/dt) is positive and -L(dI/dt) will be negative. The absolute value of the potential difference across the coil is L(dI/dt). Since the flux through the coil is increasing the coil is acting like a battery to produce an induced current in the direction opposite to the arrow in Fig. 3. Therefore, the high and low potential sides of the inductor are "a" and "b" respectively. Recall that inside a battery the current flows from low to high potential. The same thing happens in an inductor. The high and low potential sides of the resistor are "c" and "d" respectively. The current flowing in the circuit at a time "t" after the applied potential difference rises from zero to \( V_0 \) is \( I(t) \). Applying Kirchoff's Law we proceed counterclockwise around the circuit giving:

\[ \text{Separation of this equation } +V_0 -L \left( \frac{dI(t)}{dt} \right) -RI(t) = 0 \text{ the variables in this equation produces:} \]

\[ \frac{dI}{V_0} = \left( \frac{R}{L} \right) dt \]

If the left side of this equation is integrated from a zero current (at \( t = 0 \)) to \( I(t) \) (at any later time "t") and the right hand side from \( t = 0 \) to any later time "t" we obtain:

\[ I(t) = \frac{V_0}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

Experimental Details and Analysis (i):
The Induced Current when the Applied Potential Suddenly Decreases to Zero: Fig. 5 shows the Pasco RLC circuit board. In this part of the experiment we are going to observe the current as it falls from its initial value to zero.

- Measure and record the total resistance of the coil and 10-ohm resistor using the multi-meter connected between terminals "A" and "B" of Fig. 5. Note: nothing else should be connected to the circuit board.
- Use a wire to connect the ground terminal of the 850 Pasco Interface Box generator to terminal "A" of the circuit board and a second wire from the generator terminal of the box to terminal "B" of the circuit board. The output voltage of the signal generator is applied across the 10-ohm resistor and the coil in series to create the circuit shown in Fig. 3.
- In Capstone select the Hardware Setup icon. Click on the signal output icon button (which is on the extreme right side) to open a dialog box. Select the Output Voltage-Current Sensor. Under the Signal Generator icon select the 850 Output 1.

In this window select the following:
- **Positive** Square Wave
- Sweep Type off
- Frequency 8.00 Hz.
- Amplitude 4.00 V.
- Voltage Limit 5.0 V.
- Current Limit 1.50 A.

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• Signal generator set to Auto and minimize the generator window.

• Set the sample rate to 20.00 kHz.
• Create a plot window to display the **Output Current** versus time.
• Use the **Recording Conditions** button to select the following start and stop conditions. Under **Start Conditions** select **Measurement Based**, then **Output Current** then **falls below**. Enter a value of 0.25 (A) for the current. Since the total resistance of the coil and resistor is about 15 (Ω) and the applied potential is 4.00 (V) the initial current should be about 4.0/15 or 0.27 A. Choose in the same way to stop the data collection when the **Output Current** falls below 0.01 amperes. With these settings data collection will begin automatically when the current just begins to decrease and will stop automatically when the current is getting close to zero.
• Click on **Record** to obtain a data set.
• Examine the final falling edge of the data obtained for the current in the circuit. It’s probably easier to use a selection box around the data of interest then use the expand plot icon. Does the plot seem to be in qualitative agreement with the theoretical expectations of Eq. 8? Try to fit the **falling edge** of your data using the **Capstone** natural exponent fit which is: 

$$ A e^{(-Bt)} + y_0 $$

The selection box should include the points just as they fall from 0.25A to the last point before the plot intercepts the X axis. The important term is the exponent “B” which is equal to 1/τ. **However, you are may find that the calculated fit is poor.** If the above fails. You may need to enter your own equation that takes a non-zero starting position into account by using the **User Defined:** f(x) choice from the fitting menu. The easiest way is to first choose the inverse exponent fit then open the equation editor and copy only the equation near the top. Go back to the equation selection menu and choose **User Defined:** f(x). Open the equation editor and paste the copied equation into the equation bar. Edit the equation to remove the 1- term. You should be left with the following equation:

$$ Ae^{-B(t-t_o)} + C $$

Select the t_o box and enter your time offset. Find the experimental value of the time constant "τ" from "B" and then find the inductance "L" of the coil using the total resistance of the circuit which includes the 10 ohm resistor and the resistance of the coil. Find the percent error between the experimental value of the inductance and the value marked on the circuit board.

• Use the **icon** to obtain the time at which the current is reduced to (1/e) of its initial value at t = 0. Compare this result with your time constant obtained in the previous step.

• Insert the aluminum cylinder in the coil and obtain a new data set **on the same graph**. Do a natural exponent fit of the falling edge of this data and find the inductance with aluminum in the coil.
• Insert the iron cylinder and obtain a third data set **on the same graph**. From a natural exponent fit obtain the inductance with iron in the coil.

**Experimental Details and Analysis (ii): The Induced Current when the Applied Potential Suddenly Increases from Zero to V_0:**
• The data for this part of the experiment is collected in exactly the same way except for two small changes in the Start and Stop conditions. Use the **Recording Conditions icon** to select the following start and stop conditions. Under **Start Conditions** select **Measurement Based**. Select **Output Current** and then **Rises Above**. Enter 0.01 for this current. Also choose in the same way to stop the data collection when the **Output Current**, **Rises Above** a value 0.25 (A). With these settings data collection will begin automatically when the current just begins to increase and will stop automatically when it rises close to its final, maximum value.
• The material in the coil should be air. Click on **start** to obtain a data set which should resemble the rising current data shown in Fig. 4 and predicted by Eq. (9).
• Try to fit the rising edge of your data using the **Capstone** Inverse exponent fit which is:

$$ A(1 - e^{-B(t-t_o)}) + C $$

Again, if you have difficulty fitting the curve in the Curve Fit Editor make sure your time axis starts at zero where the signal of interest changes. You can use the Equation Editor and check the t_o box and enter your time offset if necessary.

Find the experimental value of the time constant "τ" from "B" (using Eq. 9) and then find the inductance "L" of the coil using the total resistance of the circuit which includes the 10 ohm resistor and the resistance of the coil. Find the percent error between the experimental value of the inductance and the value marked on the circuit board.

**Appendix: (i) Detailed Description of Our Model for the Falling Magnet:**

The magnetic field of the disc magnet can be well represented by the field of a current loop which is shown in the Fig. 1. In the following discussion the current loop replaces the disc magnet. For the assumed direction of the current “I” the magnetic field points down as shown in the first picture of Fig. 6. Two symmetrically located eddy current loops (which are inside the conducting tube) are shown. Since the magnet is moving towards the lower eddy loop the flux downwards through it is increasing and according to Lenz’s Law the induced current in the eddy loop is in the direction shown.
(CCW as seen from above) to create an induced field which is up. The flux through the upper loop is decreasing in the downward direction and the induced current in that loop is in the direction shown (CW from above) to produce an induced field which is down. The direction of these induced fields is shown in the second picture of Fig. 6. The third picture shows the resultant induced field of both eddy loops at the location of the falling magnet loop as viewed from above. The resultant field is radial outwards. The direction of the current in the coil representing the magnet is shown in the third figure. The direction of the magnetic force on a short length "ds" of the loop is obtained from the Lorentz formula:

\[ d\vec{F} = I d\vec{s} \times \vec{B} \]

From the vector cross-product we see that the direction of this force is \( \text{up} \) and therefore is a resistive type of force.

(ii) The Velocity Dependence of the Resistive Force:

We examine, using Fig. 7, the induced current in a single eddy current loop produced by the current loop that represents the falling magnet. The magnitude of the magnetic field of the magnet current loop depends on the distance "x" from the loop. The flux of the magnetic flux through the loop would be difficult to calculate exactly since, as shown in Fig (7), the direction and magnitude of the field over the eddy loop changes with position. However, the flux must be a function, \( \Phi(x) \), of the distance "x". The figure shows the position of the magnet loop at a final time “t + dt” and at an initial time “t” where “dt” is a very small, differential time interval during which the magnet moves the distance “dx”. At the initial time the distance from the magnet loop to the eddy loop is “x + dx” and at the later time the distance is “x”. Since the magnet moved a distance “dx” in a time interval “dt” the speed of the magnet is \( v= dx/dt \). Faraday’s law can be applied to find the induced \( \mathcal{E}_{\text{Emf}} \) in the eddy loop. Faraday’s Law shows that the induced \( \mathcal{E}_{\text{Emf}} \) in the eddy loop is directly proportional to the speed of the falling magnet. The induced current in the eddy loop is also proportional to the speed by Ohm’s Law. According to the Biot-Savart Law, the magnetic field created by the induced eddy current at the location of the falling magnet loop is proportional to the eddy current. Finally, the total upwards force exerted on the falling magnet current loop is directly proportional to the field of the eddy current loop. Therefore,
the net force on the magnet for a single eddy loop is directly proportional to the speed of the magnet. The total force on the falling magnet from all eddy current loops is obtained by summing over the forces created by all eddy loops. The magnitude of the total resistive magnetic force acting on the magnet is:

\[ F_{\text{ret}} = cv. \]

In this equation, ‘c’ is a constant (that would be very difficult to calculate) and “v” is the speed of the falling magnet.